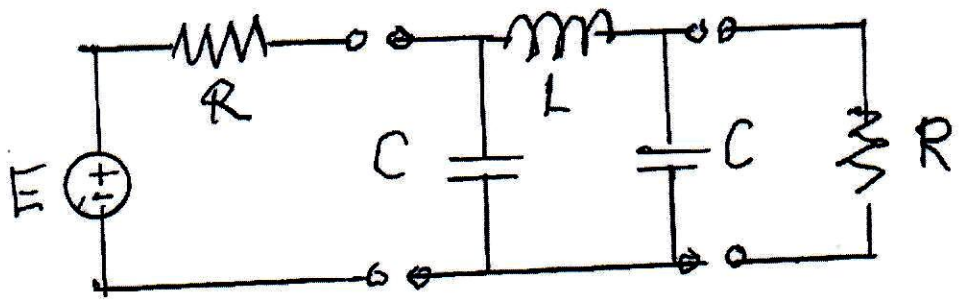
1. (a) Find the chain matrix of the two-port shown below.

(b) Find the voltage gain V_2/E if the two-port operates between two equal resistors R.



$V_1 = AV_2 - BI_2 \qquad I_1 = CV_2 - DI_2$

$A = (V_1/V_2)_{I_2=0} = \dfrac{sL + 1/sC}{1/sC} = s^2LC + 1$

$B = V_1/-I_2 \,|_{V_2=0} = sL$

$C = I_1/V_2 \,|_{I_1=0} = sC(s^2LC + 2)$

$D = I_1/-I_2 \,|_{V_2=0} = s^2LC + 1$

$AD - BC = (s^2LC+1)^2 - s^2LC(s^2LC+2) = 1$ ✓

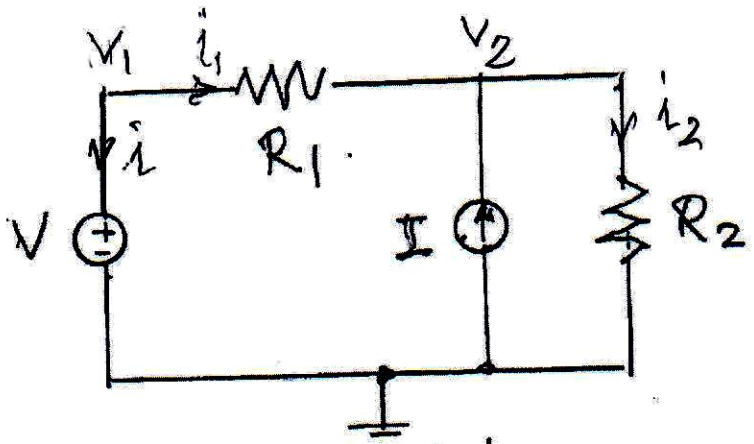
Lecture notes p. 67:

$$\frac{V_2}{E} = \frac{1}{A + B/R + CR + D}$$

$V_2/E = 1/[2(s^2LC+1) + sL/R + sRC(s^2LC+2)]$

$V_2/E = 1/[s^3RLC^2 + s^2 2LC + s(L/R + 2RC) + 2]$

2. Find all voltages and currents in the circuit below. The element values are $R_1 = 1\ \text{k}\Omega$, $R_2 = 2\ \text{k}\Omega$, $V = 3\ \text{V}$ and $I = 1\ \text{mA}$.



Note : Use MNA !

KCL1 : $(V_1 - V_2)/R_1 + i = 0$

KCL2 : $(V_2 - V_1)/R_1 + V_2/R_2 = I$

KVL : $V_1 = V$

$$\begin{bmatrix} G_1 & -G_1 & 1 \\ -G_1 & G_1+G_2 & 0 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ i \end{bmatrix} = \begin{bmatrix} 0 \\ I \\ V \end{bmatrix}$$

Solution:

$$-G_1 V_2 + i = -G_1 V$$

$$-G_1 V + (G_1+G_2) V_2 = I$$

$$V_2 = (I + G_1 V)/(G_1+G_2) = \frac{10^{-3} + 3\times 10^{-3}}{1.5 \times 10^{-3}}$$

$V_2 = 8/3\ \text{V}$, $i = +10^{-3}(8/3 - 3) = -\frac{1}{3}\ \text{mA}$

$i_2 = I - i = 4/3\ \text{mA}$ $V_2 = 8/3\ \text{V}$